On the Development of a Seligman-Style Tableau System for Hybrid Logic

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Part A

Motivation and Introduction

This is about tableau deduction in Hybrid Logic

Propositional Logic and a Very Simple Example

In ordinary propositional logic we prove the tautology $p \to (q \to p)$ by constructing a closed tableau for the negation:

$$
\begin{array}{cc}
\neg\big(p \to (q \to p)\big) & \\
p & (\neg \to) \\
\neg(q \to p) & (\neg \to) \\
q & (\neg \to) \\
\neg p & (\neg \to) \\
\times &
\end{array}
$$

One valuation is associated with a whole branch.

How About Tableaus for Modal Logic?

One good answer is the use of labels (Fitting 1983). With the labelling technique we can prove $\Diamond p \to \Diamond(p \vee q)$ to be K-valid:

1	$\neg(\Diamond p \to \Diamond(p \vee q))$	
1	$\Diamond p$	$(\neg \to)$
1	$\neg\Diamond(p \vee q)$	$(\neg \to)$
1.1	p	$(\Diamond)$
1.1	$\neg(p \vee q)$	$(\neg\Diamond)$
1.1	$\neg p$	$(\neg\vee)$
1.1	$\neg q$	$(\neg\vee)$
	$\times$	

A branch may relate to several worlds, that is, 'several' valuations.

Let's introduce Hybrid Logic

Introducing Hybrid Logic (1/2)

Hybrid logic is like orthodox modal logic with just a little extra.

First of all, there are *two sorts* of propositional symbols:

- Nominals: $i, j, k, \ldots$
  $\longrightarrow$ These are true at exactly one world.
- Ordinary propositional symbols: $p, q, r \ldots$.

Introducing Hybrid Logic (2/2)

Second of all, we can express satisfaction of formulas.
We have the satisfaction operator:

- $@_i$

Thus, $@_i\varphi$ claims that φ is satisfied at the world named by i;

$$\mathfrak{M}, w \models @_i\varphi \quad \text{iff} \quad \mathfrak{M}, w' \models \varphi,$$

where w' is the denotation of i.

The formulas of our hybrid logic are generated by:

$$\varphi ::= i \mid p \mid \neg\varphi \mid \varphi \to \psi \mid \Diamond\varphi \mid @_i\varphi.$$

Hybrid Logic is Expressive

Using nominals, accessibility can be expressed:

$$\Diamond i$$

Frames can be defined:

$i \rightarrow \Diamond i$	Reflexivity
$\Diamond\Diamond i \rightarrow \Diamond i$	Transitivity
$i \rightarrow \neg\Diamond i$	Irreflexivity
$@_j \Diamond i \vee @_j i \vee @_i \Diamond j$	Trichotomy

And many more...

Internalizing Labelled Deduction

1	$\neg(\Diamond p \to \Diamond(p \vee q))$	$@_j \neg(\Diamond p \to \Diamond(p \vee q))$	
1	$\Diamond p$	$@_j \Diamond p$	$(\neg \to)$
1	$\neg\Diamond(p \vee q)$	$@_j \neg\Diamond(p \vee q)$	$(\neg \to)$
		$@_j \Diamond i$	$(\Diamond)$
1.1	p	$@_i p$	$(\Diamond)$
1.1	$\neg(p \vee q)$	$@_i \neg(p \vee q)$	$(\neg\Diamond)$
1.1	$\neg p$	$@_i \neg p$	$(\neg\vee)$
1.1	$\neg q$	$@_i \neg q$	$(\neg\vee)$

In the latter tableau the 'world-handling' is completely internalized (Blackburn, 2000).

Questions

- Is *all* the labelling machinery done by the @ really necessary?
- Is the labelling approach the only feasible approach to hybrid tableaus?
- How about "Rules for All"? (Seligman 1997)
- Is there some way to distinguish between 'the view from nowhere' (the global) and 'the view from now and here' (the local)?

$$@_j \neg(\Diamond p \to \Diamond(p \vee q))$$
$$@_j \Diamond p$$
$$@_j \neg\Diamond(p \vee q)$$
$$@_j \Diamond i$$
$$@_i p$$
$$@_i \neg(p \vee q)$$
$$@_i \neg p$$
$$@_i \neg q$$

Now we turn to Seligman-style tableaus

Introducing Seligman-Style Tableaus

General idea: Chop up the branches into blocks. Such blocks are partial descriptions of particular worlds.

Our example:

$\neg(\Diamond p \to \Diamond(p \vee q))$	
$\Diamond p$	$(\neg \to)$
$\neg\Diamond(p \vee q)$	$(\neg \to)$
$\Diamond i$	$(\Diamond)$
$@_i p$	$(\Diamond)$
i	GoTo
p	$(@)$
$\neg(p \vee q)$	$(\neg\Diamond)$
$\neg p$	$(\neg\vee)$
$\neg q$	$(\neg\vee)$

The Rest of This Talk

Part B: The basic tableau system and results

Part C: Future work

Part D: Conclusion

Part B

The basic tableau system and results

Tableau Rules: The Propositional Part

Propositional rules are simply preserved unchanged from the propositional calculus:

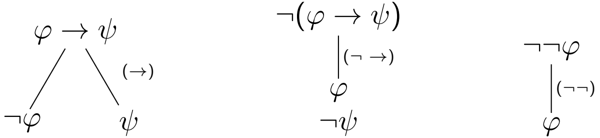


Tableau Rules: Hybrid Extension (1/2)

Seligman-style rules

$$\frac{\Diamond\varphi}{\Diamond i \quad @_i\varphi}\ (\Diamond),\ i \text{ fresh}$$

$$\frac{\neg\Diamond\varphi \quad \Diamond i}{\neg @_i\varphi}\ (\neg\Diamond)$$

$$\frac{i \quad @_i\varphi}{\varphi}\ (@)$$

$$\frac{i \quad \neg @_i\varphi}{\neg\varphi}\ (\neg @)$$

Labelled rules

$$\frac{@_j\Diamond\varphi}{@_j\Diamond i \quad @_i\varphi}\ (\Diamond),\ i \text{ fresh}$$

$$\frac{\neg @_j\Diamond\varphi \quad @_j\Diamond i}{\neg @_i\varphi}\ (\neg\Diamond)$$

$$\frac{@_i @_j\varphi}{@_j\varphi}\ (@)$$

$$\frac{\neg @_i @_j\varphi}{\neg @_j\varphi}\ (\neg @)$$

The labelled rules are slightly modified rules from (Blackburn 2000).

Tableau Rules: Hybrid Extension (2/2)

Seligman-style rules

$$\frac{}{i}\ \text{(GoTo), } i \text{ on branch}$$

$$\Big|\ \text{(Name), } i \text{ fresh} \qquad i$$

$$\frac{\begin{array}{c}\varphi \\ i\end{array}}{\begin{array}{c}\vdots \\ i \\ \big|\ \text{(Nom)} \\ \varphi\end{array}}$$

Labelled rules

$$\Big|\ \text{(Ref)} \qquad @_i i$$

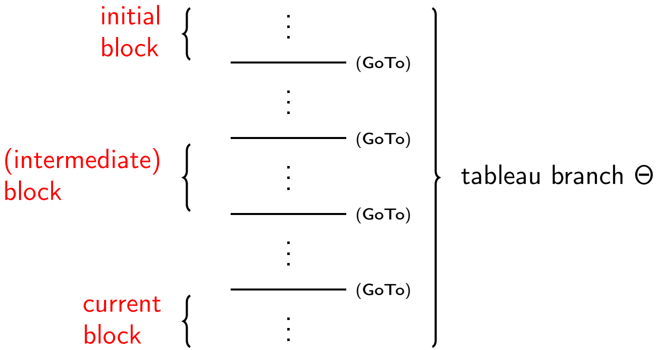
$$\begin{array}{c} @_i j \\ @_i \varphi \\ \big|\ \text{(Nom1)} \\ @_j \varphi \end{array} \qquad \begin{array}{c} @_i j \\ @_i \Diamond k \\ \big|\ \text{(Nom2)} \\ @_j \Diamond k \end{array}$$

In Nom1 φ is propositional symbol or nominal.

Chopping up in Blocks (1/2)

1	$\neg(\Diamond @_i p \to @_i p)$	
2	$\Diamond @_i p$	$(\neg \to)$ on 1
3	$\neg @_i p$	$(\neg \to)$ on 1
4	$\Diamond j$	$(\Diamond)$ on 2
5	$@_j @_i p$	$(\Diamond)$ on 2
6	j	GoTo
7	$@_i p$	$(@)$ on 5,6
8	i	GoTo
9	$\neg p$	$(\neg @)$ on 3,8
10	p	$(@)$ on 7,8
	$\times$	closure by 9, 10

Chopping up in Blocks (2/2)



The opening nominals are special. Together with the block structure they play the role that the outermost @ play in the labelled calculus. This is our externalisation.

Results for the Basic System

The tableau-rules are sound: Satisfiability is preserved blockwise.

By providing a translation from the labelled calculus into the Seligman calculus we can prove:

Theorem 1. The Seligman calculus is complete.

By imposing restrictions on the Seligman calculus and by providing a translation from this restricted calculus into a terminating labelled calculus we can prove:

Theorem 2. A restricted Seligman version of the calculus is terminating, but still complete.

Part C

Future work

Extensions of the Basic System

The basic logic can be extended with $\downarrow$ and/or the universal modality A. There are natural non-labelled rules.

For first-order hybrid logic over constant domains we have also developed a system; with the ordinary first-order rules:

$$\dfrac{\exists x \varphi(x)}{\varphi(b)}\ (\exists) \qquad \dfrac{\neg\exists x \varphi(x)}{\neg\varphi(t)}\ (\neg\exists) \qquad \dfrac{}{t = t}\ (\text{Ref}) \qquad \dfrac{t = s \quad \varphi(t)}{\varphi(s)}\ (\text{RR})$$

+ one more rule, if one wants to make use of the extra expressiveness given the combination of first-order-logic and hybrid logic.

Other things to Look at

Moreover, we plan to look at

- Semantic completeness proofs for these systems, and
- Cut elimination

Part D

Conclusion

Coming Back to our Research Questions

- Is *all* the labelling machinery done by the @'s really necessary?
  $\longrightarrow$ No, we can externalise some of them.
- Is the labelling approach the only feasible approach to hybrid tableaus?
  $\longrightarrow$ No.
- How about "Rules for All"? (Seligman 1997)
  $\longrightarrow$ Good idea!
- Is there a way to distinguish between 'the view from nowhere' (the global) and 'the view from now and here' (the local)?
  $\longrightarrow$ Yes, the Seligman calculus makes that possible.