A.N. Prior's Analysis of the Ontological Argument

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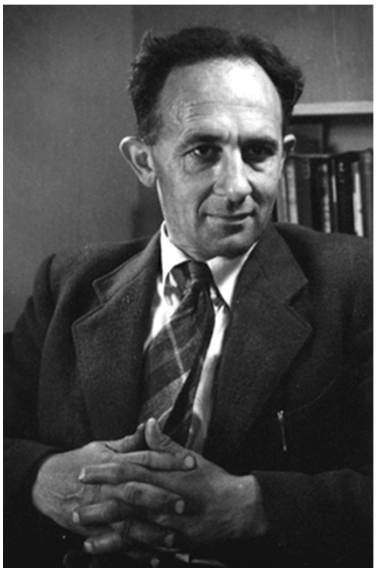


Centenary conference in Oxford, August 21-22, 2014

Arthur Norman Prior (1914–69)

http://www.prior.aau.dk

Prior often worked with Anselm’s so-called ontological argument for the existence of God:

- Published sections in his books and papers
- At least three unpublished papers
- Discussions in his letters (Jack Smart, von Wright)

His motivation:
- Personal interest in the topic
- Strong interest in formalization
- Conceptual relations to the notions of existence
- Conceptual relations to the notions of modality

Only if there is a God, I ought to go to church.
I ought to go to church.
Therefore:
There is a God.

<u>Meta-theorem:</u>
For an argument to constitute a proof it must not only be formally valid; it is requisite that those to whom it is adressed should be convinced of the truth of its premisses, at all event more convinced of the truth of its premisses than of the falsehood of its conclusion.

Real existence is a perfection
Some thinkable has all perfections
Therefore:
Something that has all perfections has real existence.

Real existence is a perfection
Therefore:
Whatever has all perfections has real existence

To say that a man believes in God, for example, may be to say that God exists in his belief-world (cf. Anselm's talk of God existing 'in the mind'), and this in turn to characterise his belief-world as 'theistic'. Perhaps in another man's belief-world God does *not* exist, i.e. that belief-world is 'atheistic'. ["Modal Logic and the Logic of Applicability ", 1968]

e: “real existence”
P: “is a perfection”
T: “is a thinkable”
H: “has”

(1) Pe
(2) ∃x (Tx & ∀y(Py → Hxy))
(3) ∀x (∀y(Py → Hxy) → Hxe); from (1)
(4) ∃x (Tx & ∀y (Py → Hxy) & Hxe); from (2) and (3)
(5) ∃x (∀y(Py → Hxy) & Hxe), from (4).

Here existence is represented as a subject, e.

P: “is perfect”
T: “is a thinkable”
R: ”is really existing”

(1) ∀x (Px → Rx)
(2) ∃x (Tx & Px)
(3) ∃x (Tx & Px & Rx), from (1) and (2)
(4) ∃x (Px & Rx), from (3)

Here existence is represented as a predicate, R.

P: “is perfect”
T: “is a thinkable”
R: ”is really existing”
$Px \equiv Dx \,\&\, Rx$, where D is the conjunction of the remaining components of perfection,

(1) $\forall x\,(Dx \,\&\, Rx \rightarrow Rx)$
(2) $\exists x\,(Tx \,\&\, Dx \,\&\, Rx)$
∴ (3) $\exists x\,(Dx \,\&\, Rx)$

(1) is a tautology and can now be dropped:
(1) $\exists x\,(Tx \,\&\, Dx \,\&\, Rx)$
∴ (2) $\exists x\,(Dx \,\&\, Rx)$

Real existence is a perfection
Some thinkable has all perfections
Therefore:
Something that has all perfections has real existence.

A more convincing version?

Real existence is a perfection
It is thinkable that something has all perfections
Therefore:
Something that has all perfections has real existence.

To say that a man believes in God, for example, may be to say that God exists in his belief-world (cf. Anselm's talk of God existing 'in the mind'), and this in turn to characterise his belief-world as 'theistic'. Perhaps in another man's belief-world God does *not* exist, i.e. that belief-world is 'atheistic'. ["Modal Logic and the Logic of Applicability ", 1968]

Two kinds of existence

God exists in a belief-world: $\sum x\ Gx$

God exists in reality: $\exists x\ Gx$

$\exists x\ Gx \equiv \sum x\ (Gx\ \&\ Rx)$

Real existence is a perfection

It is thinkable (possible) that something has all perfections

Therefore:

Something that has all perfections has real existence.

$$(1)\ Px \equiv (Dx\ \&\ Rx)$$

$$(2)\ M(\exists x\ Px)$$

$$\therefore\quad (3)\ \sum x\ Dx,$$

Some hold this argument to be valid because they assume:

$$M(\exists xFx) \rightarrow \sum xFx$$

The following is problematic:

$$M(\exists x Fx) \rightarrow \sum x Fx$$

Assume that Fx is $\forall y(x = y)$

It can be imagined that there is only one object in the universe, i.e. $M\exists x\forall x\ (x = y)$.

But there is no possible-object (if we are going in for those things) such that every real object is identical with that possible-object.

We might say that while it is a contingent fact that, for example, lions exist, since there is nothing about the concept of lionhood which necessitates its exemplification, it is a necessary fact that there is a God, since there is something about the concept of deity which necessitates it exemplification.

”Is Necessary Existence Possible? ”,
Philosophy and Phenomenological Research,
Vol. 15, No. 4, (Jun., 1955), pp. 545-547

In certain well-known arguments for the existence of God, what is in fact shown is that the supposition that God could exist but does not necessitates something incompatible with itself, and so is necessarily false.

(1) Mp → LMp	(S5)
(2) (Mp → p) → (Mp → p)	(PC)
(3) L(Mp → p) → (LMp → Lp)	(S5, 2)
(4) L(Mp → p) → (Mp → Lp)	(1, 3)

Formal Logic (1955), p. 201

God is an entity whose possibility necessarily implies its existence.
God’s existence is possible
Therefore:
God’s existence is necessary.

(1) Mp → LMp	(S5)
(2) (Mp → p) → (Mp → p)	(PC)
(3) L(Mp → p) → (LMp → Lp)	(S5, 2)
(4) L(Mp → p) → (Mp → Lp)	(1, 3)

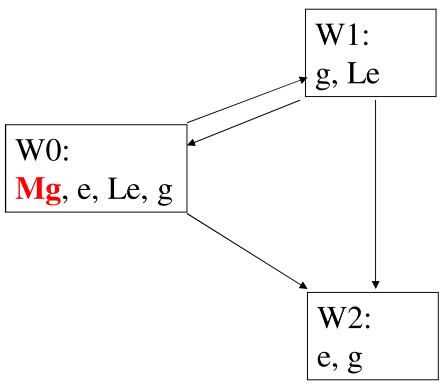
Formal Logic (1955), p. 201

(1) Mp → LMp (S5)
(2) (Mp → p) → (Mp → p) (PC)
(3) L(Mp → p) → (LMp → Lp) (S5, 2)
(4) L(Mp → p) → (Mp → Lp) (1, 3)
Formal Logic (1955), p. 201

Prior argues that (4) is not provable in S4.

If it were, then it would also be provable that:
L(MMp → Mp) → (MMp → LMp)
L(Mp → Mp) → (Mp → LMp)
Mp → LMp

Alvin Plantinga:
g(x): x has maximal greatness
e(x): x has maximal excellence
g(x) = Le(x)



S5:
L(p ⊃ q) ⊃ (Lp ⊃ Lq)
Lp ⊃ p
Lp ⊃ LLp
MLp ⊃ Lp
L= ~M~

Barcan's formula:
M(∃x: p) ⊃ ∃x: Mp

Priorean proof:
M(∃x:g(x))
∃x:Mg(x)
∃x:MLe(x)
∃x:Le(x)
∃x:g(x)