

CARNAP'S MODAL LOGIC
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Why did Carnap want to develop a modal logic? [Extracts A-E]

(1) $M \models \Box\alpha$ iff $M' \models \alpha$ for every PC model M' .

Call a model of this kind a *C-model*, and say that a wff is *C-valid* iff it is true in every C-model. If α is S5-valid then α is C-valid.

(2) $\sim\Box p$ is C-valid (because p is not true in every PC model.) But $\sim\Box p$ is not a theorem of S5.

A wff α is QC-valid iff, for every wff β obtained from α by uniform substitution for the variables of α , β is C-valid.

(3) α is QC-valid iff α is S5-valid.

Let M^+ be a class of PC models. (3) is replaced by

(4) $M \models \Box\alpha$ iff $M \models \alpha$ for every $M' \in M^+$

Say that a wff α is valid in M^+ iff $M \models \alpha$ for every $M \in M^+$, and say that α is M^+ -valid iff α is valid in every M^+ . Kanger 1957, Hintikka 1957, Kripke 1959, Bayart 1958, 1959 or Montague 1960. α is M^+ -valid iff α is S5-valid.

Carnap: Which of Lewis's modal logics is the 'correct' one.

Quine's attack on analyticity:

While Quine can accept that Carnap may have defined a formal concept which he calls 'analyticity in L ', where L is a formal language, Quine cannot accept that there is any intuitive natural language concept which can be the explicandum. (Carnap 1963, p. 919.) Carnap actually agrees that it is necessary to provide such if his account of analyticity is to have any genuine content. But he claims that it is not difficult to shew that one can be given. He alludes to the fact that empirical linguists have little trouble in making claims about the meanings of words and phrases: [Extract F]

He points us to Carnap 1955. [Extract G], and follows this up on p. 920 of Carnap 1963. [Extract H]

Can meaning postulates (Carnap 1952) lead to a modal semantics?

- (5) If Jack is a bachelor, then Jack is unmarried
- (6) All ravens are black

Up to the theorist to stipulate which sentences are to be regarded as analytic. That is why Carnap had to take up Quine's challenge to give an empirical criterion for analyticity, because he has to be able to distinguish a theorist who stipulates (9) as *analytic* from one who requires it merely to be true.

Quine's complaints about 9 and the number of planets:

- (7) 9 is necessarily greater than 7
- (8) The number of planets = 9
- \therefore (9) The number of planets is necessarily greater than 7

Carnap 1946:

- (i) Individuals are represented by individual constants and predicates apply to them (via state descriptions)
- (ii) Quantifiers range over individuals
- (ii) $\forall x\alpha$ is true in a state description iff $\alpha[c/x]$ is true in that state description for every constant c replacing x

Quine's argument in primitive notation:

- (10) $\Box\text{odd } 9$
- (11) $\exists x(\text{number of planets } x \wedge x = 9)$

Conclusion: Three possibilities:

- (12) $\Box\exists x(\text{number of planets } x \wedge \text{odd } x)$
- (13) $\exists x\Box(\text{number of planets } x \wedge \text{odd } x)$
- (14) $\exists x(\text{number of planets } x \wedge \Box\text{odd } x)$

Quine characterised essentialism as 'the adoption of an asymmetrical attitude to different ways of specifying the same object'. (Smullyan 1947 scope.)

- (15) $\forall x(\text{odd } x \supset \Box\text{odd } x)$
- (16) $\forall x(\text{number of planets } x \supset \Box\text{number of planets } x)$.

The theorist is saying, 'by ϕ I mean a predicate which as a matter of meaning I define as, among other things, applying to a '.

Quine characterises essentialism as privileging certain ways of referring to an individual, but there seems no problem in construing a language-user as referring to a particular *thing*, and as claiming that a predicate ϕ is being used in such a way that it has to apply to that *thing*.

Carnap 1947:

- (i) Individuals are represented by individual constants and predicates apply to them (relative to state descriptions) *Individual concepts* (aka *intensional objects*) are functions from state-descriptions to constants.
- (ii) Quantifiers range over individual concepts
- (iii) $\forall x\alpha$ is true in a state description iff $\alpha[c/x]$ is true in that state description for every individual concept i where c is the value of i in that state description

Carnap is clear that the quantifiers range over *all* individual concepts, not just those expressible in the language.

$$\Box\exists x\text{number of planets } x \supset \exists x\Box\text{number of planets } x$$

Quine's complaint at this point (Quine 1953a, p. 152f) is that Carnap replaces the domain of things as the range of the quantifiers with a domain of individual concepts.

A
For me personally, Wittgenstein was perhaps the philosopher who, besides Russell and Frege, had the greatest influence on my thinking. The most important insight I gained from his work was the conception that the truth of logical statements is based only on their logical structure and on the meaning of the terms. Logical statements are true under all conceivable circumstances; thus their truth is independent of the contingent facts of the world. On the other hand, it follows that these statements do not say anything about the world and thus have no factual content. (Autobiography: Carnap 1963, p. 25)

B
I discovered that in these questions, even though my thinking on semantics had originally started from Tarski's ideas, a clear discrepancy existed between my position and that of Tarski and Quine, who rejected the sharp distinction I wished to make between logical and factual truth. (Carnap, 1963, p. 36)

C
Some of those who accept the semantical concept of truth reject a sharp distinction between logical and factual truth. Most prominent among them are Tarski and Quine. During the academic year 1940-41, when all three of us were at Harvard, we discussed this problem in great detail. They believed that, at best, a distinction of degree could be made. At that time I gave a talk on the relation of mathematics to empirical science in a large discussion group of faculty members interested in the foundations of science. My main thesis was that mathematics has no factual content and, therefore, is not in need of empirical confirmation, but that it nevertheless has a very important function in empirical science as an instrument of deduction. I thought that this was an old story and at any rate a purely academic question. But to my great surprise, the audience responded with vehement emotions. Even before I had finished my lecture, excited objections were raised. Afterwards we had a long and heated discussion in which several people often talked at the same time. Richard von Mises stated bluntly that the sentence " $2 + 2 = 4$ " (if taken, not as a theorem in an uninterpreted axiom system, but in its customary interpretation) was just as much of an empirical nature as the sentence "Solid bodies expand when heated". I thought: are we now back with John Stuart Mill? The attacks by Tarski and Quine were even more spirited, but also more discerning. Many others rejected my view. I think Feigl was the only one who clearly shared my position. But, on the whole, the discussion was too vehement to permit a good mutual understanding. (Carnap, 1963, p. 64f)

D
But the decisive point is the following: *in order to determine whether or not one sentence is a consequence of another, no reference need be made to the meaning of the sentences. The mere statement of the truth-values is certainly too little; but the statement of the meaning is, on the other hand, too much. It is sufficient that the syntactical design of the sentences be given.* (Carnap 1937, p. 258. The italics are Carnap's)

E
After defining semantical concepts like logical truth and related ones, I proposed to interpret the modalities as those properties of propositions which correspond to certain semantical properties of the sentences expressing the propositions. For example, a proposition is logically necessary if and only if a sentence expressing it is logically true. (Carnap 1963, p. 62)

F
It seemed rather plausible to me from the beginning that there should be an empirical criterion for the concept of the meaning of a word or phrase in view of the fact that linguists traditionally determine empirically the meanings, meaning differences, and shifts of meanings of words, and that with respect to these determinations they reach a measure of agreement among themselves which is often considerably higher than that reached for results in most of the other fields of the social sciences. Quine's arguments to the effect that the lexicographers actually have no criterion for their determinations did not seem at all convincing to me. (Carnap 1963, p. 919f)

G
I shall now plead for the intensionalist thesis. Suppose, for example, that one linguist, after an investigation of Karl's speaking behavior, writes into his dictionary the following :

(1) *Pferd* horse,

while another linguist writes:

(2) *Pferd* horse or unicorn.

Since there are no unicorns, the two intensions ascribed to the word 'Pferd' by the two linguists, although different, have the same extension. If the extensionalist thesis were right, there would be no way for empirically deciding between (1) and (2). Since the extension is the same, no response by Karl, affirmative or negative, with respect to any actual thing can make a difference between (1) and (2). But what else is there to investigate for the linguist beyond Karl's responses concerning the application of the predicate to all the cases that can be found? The answer is, he must take into account not only the actual cases, but also possible cases. The most direct way of doing this would be for the linguist to use, in the German questions directed to Karl, modal expressions corresponding to 'possible case' or the like. To be sure, these expressions are usually rather ambiguous; but this difficulty can be overcome by giving suitable explanations and examples. I do not think that there is any objection of principle against the use of modal terms. On the other hand, I think that their use is not necessary. The linguist could simply describe for Karl cases, which he knows to be possible, and leave it open whether there is anything satisfying those descriptions or not. He may, for example, describe a unicorn (in German) by something corresponding to the English formulation: 'a thing similar to a horse, but having only one horn in the middle of the forehead'. Or he may point toward a thing and then describe the intended modification in words, e.g.: 'a thing like this one but having one horn in the middle of the forehead'. Or, finally, he might just point to a picture representing a unicorn. Then he asks Karl whether he is willing to apply the word 'Pferd' to a thing of this kind. An affirmative or a negative answer will constitute a confirming instance for (2) or (1) respectively. This shows that (1) and (2) are different empirical hypotheses. (Carnap 1955, p. 238)

H
Let us suppose that two linguists study the natural language L as used by the person X . Let us suppose that L consists of some English words and English sentences, among them the following sentence:

(S_1) 'All ravens are black'.

We assume that the two linguists agree on the basis of previous experiments that X uses the words 'all' and 'are' in the ordinary sense, and that X has repeatedly affirmed the sentence S_1 ; and hence presumably regards it as true. Now the first linguist states the following hypothesis:

(5) 'The sentence S_1 is analytic in language L for person X .'

The other linguist denies this hypothesis. In order to obtain evidence relevant for (5), the linguists say to X : 'Mr. Smith told us that he had found a raven which is not black but white, and that he will show it to you tomorrow. Will you then revoke your assertion of S_1 ?' Let us consider the following two of many possible responses by X :

(6) 'I would never have believed that there are white ravens; and I still do not believe it until I see one myself. In that case I shall, of course, have to revoke my assertion'.

(7) 'There cannot be white ravens. If a bird is not black, then I just would not call it a raven. If Mr. Smith says that his raven is not black, then (assuming that he is not lying or joking) his use either of the word 'raven' or of the word 'black' must be different from my use'.

It seems obvious to me that a response like (6) would be disconfirming evidence for hypothesis (5), while a response like (7) would be confirming evidence for it. Thus it is clear that (5) is an empirical hypothesis which can be tested by observations of the speaking behavior of X . (Carnap 1963, p. 920)

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