

Medieval Modal Logic: What Is It, Where Can You Find It, And What Can You Do With It Once You've Got It

Dr. Sara L. Uckelman

sara.uckelman@asia-europe.uni-heidelberg.de

Karl Jaspers Centre for Advanced Transcultural Studies
Ruprecht-Karls Universität Heidelberg

Modality and Modalities 2014
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What Is It?

A naive view:

Logic in the Middle Ages

Modal Logic in the Middle Ages

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Logic in the Middle Ages
The syllogistic

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Modal Logic in the Middle Ages
The modal syllogistic

But there's *so much more*

Medieval logic is:

- A linguistic science: Because modalities are everywhere in natural language, they are everywhere in medieval logic.
- Deeply rooted in pedagogical practice: Because medieval pedagogy was disputational in nature, medieval logic is rife with dynamics and interaction.

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- Deeply rooted in pedagogical practice: Because medieval pedagogy was disputational in nature, medieval logic is rife with dynamics and interaction.

... a far cry from the stultifying, boring, limiting, finite syllogistic!

A lightning tour of:

- Alethic modalities
- “Theological” modalities
- Action modalities
- Epistemic modalities

with a few brief mentions of deontic modalities.

Alethic modalities

Necessary, Possible, Impossible, Contingent,
but also: True, False.

- Rules for semantic ascent and descent (very restricted).
- Distinction between modalities as operators and modalities as predicates: “It is true that ϕ ” vs. “ ϕ is true”.
- Distinction between divided and composite reading of sentences with modalities: “Necessarily a is F ” vs. “ a is necessarily F ”.
- The alethic modal syllogistic.
- Modal and non-modal definition of logical consequence; consequence *per se* and consequence *per accidens*.
- Modal consequences.

“Theological” modalities (1)

- God’s power and its relationship to possibility
- Impossible *positio*
- Paralogisms of the trinity / modes of predication

God's power and its relationship to possibility

- (Modern) Can God make a stone so heavy he couldn't lift it?

God's power and its relationship to possibility

- (Modern) Can God make a stone so heavy he couldn't lift it?
(Medieval) Can God make it that "A human being is a donkey" is true?
- Buridan's sophism, "No proposition is negative, therefore some proposition is negative", and the distinction between "possible" and "possibly-true".

Impossible *positio*

Positio: A type of disputation between an Opponent and a Respondent, generally beginning from a false premise.

- Respondent must concede every proposition equivalent to or following from what he has already conceded along with the negations of things he's already denied.
- Respondent must deny every proposition contradictory to or whose negation follows from he has already conceded along with the negations of things he's already denied.
- For any other proposition, Respondent must concede if he knows it is true, deny if he knows it is false, and doubt if he is uncertain.

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Impossible *positio* starts from a premise which is impossible, but only *per accidens* and not *per se*: A grading of types of (im)possibility.



Paralogisms of the Trinity

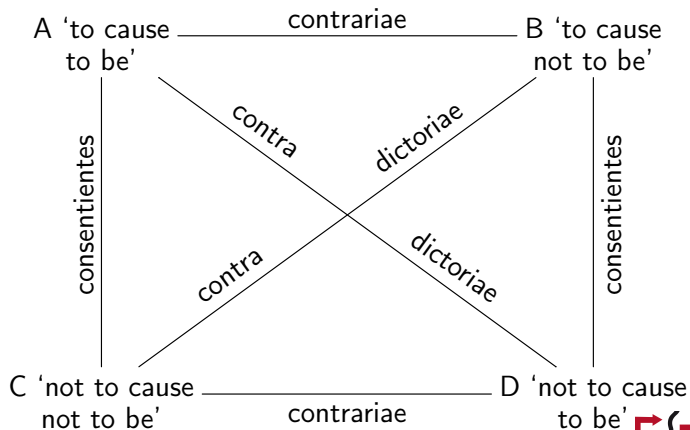
God is the Father
God is the Son
 But the Father is not the Son

Explaining why these apparently valid syllogisms have true premises but a false conclusion was an important task for medieval theologians.

Action modalities

- Public announcements and disputational commitments: When you are in a public debate, how do your actions (public announcements) affect your future commitments in the debate?
- Anselm of Canterbury on 'to do' (he also talks about 'to will' and 'to desire'): The first to recognize the modal nature of agency.

Anselm on 'to do'



Epistemic modalities

- Theories of knowledge: What are the necessary and sufficient conditions for knowledge? When can you infer a statement with an epistemic operator from one which does not, and vice versa?
- Interaction of knowledge, belief, consideration: How are these related?
- Uncertainty and doubt: What are they, and how do they relate to knowledge?

Gaetanus on the four ways to use 'to know'

- ① *Communiter*: 'to believe firmly, without hesitation, together with the fact that it is so'.
- ② *Proprie*: 'a mental grasp of anything true and necessary without a danger that the opposite be the case'.
- ③ *Magis proprie*: 'a mental grasp of anything by means of a demonstration, be it *demonstratio-quia* or *demonstratio-propter quid*, be it universal or particular'.
- ④ *Propriissime*: 'a mental grasp of anything acquired by a most powerful demonstration, which is in some way different from a *quia*-demonstration'.

Note the important occurrence of the concept of *notitia* 'mental grasp'.

Where Can you Find it?

Two types of answers

- 1 The theoretical: The context of logic and where it was practiced/developed in the Middle Ages; the general specific types/genres of texts to look into for specific types of modalities.
- 2 The practical: Some concrete suggestions for those who don't read Latin very well.

The context of logic in the Middle Ages

- Undergraduate curriculum
- Graduate research and teaching
- 1277 condemnation at Paris.

The types of sources to look at

Alethic modalities:

- Treatises on consequences
- Commentary on the *Prior Analytics* (for the syllogistic)
- Introductions to logic (especially in the 13th C)

The types of sources to look at

Theological modalities:

- Treatises on impossible *positio*.
- Quodlibetal disputations.
- Commentaries on the *Sentences* of Peter Lombard.

(Also to be found here: deontic modalities).

The types of sources to look at

Action modalities:

- Anselm of Canterbury's *Philosophical Fragments* and *De grammatico*.
- Treatises on *Obligationes*

(Also to be found here: deontic modalities).

The types of sources to look at

Epistemic modalities:

Ivan Boh on the “four phases”

- Considerations of the nodes ‘to know’, ‘to believe’, ‘to doubt’; Ockham and Pseudo-Scot consider how Aristotle’s modal syllogistic could have been extended to take into account these modes.
- Commentaries on the *Posterior Analytics* and Aristotle’s view of demonstrative knowledge.
- The search for necessary and sufficient conditions for knowing contingent propositions; problems with iteration of epistemic modalities.
- The incorporation of epistemic modalities into theories of consequence.

The types of sources to look at

Epistemic modalities:

- Treatises on knowledge and uncertainty: *De scire et dubitare*, Paul of Venice, William Heytesbury.
- Commentaries on the *Posterior Analytics*.
- Treatises on consequences.
- The genre of *obligationes* called *dubitatio*.

The practical side

There is a wealth of material on medieval logic; much is still available online in manuscript or early printed edition. Of the material that has been critically edited in modern times, only a percentage of that has been translated into English (or German). Nevertheless, good places to start if you don't read Latin:

- Ockham's *Summa Logicae* (Also: His treatise on predestination, for temporal modalities.)
- Buridan's *Summulae de dialecticis* (The section on *Sophismata* argues for an interval-based semantics for temporal logic.)
- Paul of Venice's *Logica Magna* (translation incomplete)
- Kilvington's *Sophismata*
- Sherwood's *Syncategoremata* (German only); *Introductiones ad logicam* (English and German)
- *Cambridge Translations of Medieval Philosophical Texts*



What Can You Do With It?

- Modal approaches to agency.
- Different grades of possibility.
- Imperfect agents: Uncertainty, logical non-omniscience.
- Interaction of dynamics and (disputational) obligations.

(Some of) What *I've* Done With It

- A syllogistic system for reasoning about the Trinity.
- A formal system for *obligationes*.
 - ▶ Basic
 - ▶ Higher-order
- Lying and deceit
- Questions of uncertainty.
- Scope distinctions without quantifiers

A syllogistic system for reasoning about the Trinity

An anonymous late 14th/early 15th C MS provides an extension of Aristotelian categorical syllogistic which allows one to reason—without paralogisms—about the Trinity. The trick: Distinguish different ‘modes of predication’:

- Essential
- Formal
- Personal

Only essential identity is an equivalence class; formal and personal identity are reflexive and symmetric but not transitive.

In my dissertation I develop a formal model of the rules and constraints which the medieval author articulates which completely accounts for all types of predication statements between the various components of the trinity.



A formal system for *obligationes*

Kripke frames + Epistemic logic + a public-announcement-like operator.

- Tracks whether propositions are ‘relevant’ (follow from or are contradictory to the Respondent’s commitments)
- Explain how real-world knowledge enters the debate.
- “Concede ϕ ” is equivalent to announcing ϕ .
- “Deny ϕ ” is equivalent to announcing $\neg\phi$.

General, flexible framework allows for the study of many different types of *obligationes* in the same framework.

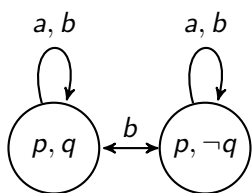
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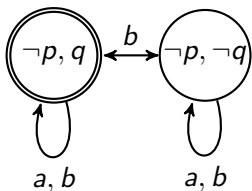
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- ... except that these announcements need not be truthful.

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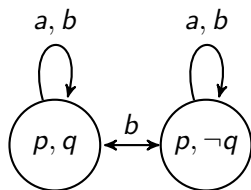
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Higher-order obligations

One particularly interesting aspect of *obligationes* is when the examples under discussion involve propositions about the Respondent's obligations in the disputation. Because the disputations are dynamic, what Respondent ought to concede depends not only on the model as it is now, but also on its history:

$$\mathfrak{M} \upharpoonright \Gamma_{n-1}, w \models O_R C \phi_n \quad \text{iff} \quad \text{either } \mathfrak{M} \upharpoonright \Gamma_{n-1} \models \phi$$

This obligation is *knowledge-based*, *history-based*, and *action-* (rather than proposition-) *oriented*.

Lying and deceit

In *positio*, Respondent is obliged concede something false. In contrast, in *dubitatio*, he is obliged to doubt something he knows to be true or knows to be false.

- Much of contemporary epistemic literature doesn't deal with lying or deceitful agents, but that's exactly what's happening in *dubitatio*: Respondent pretends he doesn't know something he in fact does.
- Such cases needn't be pernicious (surprise party).
- Connections to the logic of lying, which has recently become of interest.

What is uncertainty

Paul of Venice: “Whether something unknown to someone is uncertain to him.”

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A counterexample:

Suppose you firmly and unwaveringly believe, as you do in fact, that Antichrist will come; and I suppose further that no Antichrist will come.

Paul argues that you are certain about the proposition ‘Antichrist will come’ even though you do not know that it is true (because it is false) nor do you know it is false (in which case you would not be certain that it is true).



Scope distinctions without quantifiers

Paul of Venice again:

(1) I assume (a) that you know that A is one of the two propositions 'God exists' and 'A human being is a donkey', and (b) that one A is every A, and (c) that it is hidden from you which of the propositions is A, but (d) you know perfectly well that the proposition 'God exists' is necessary and the other, 'A human being is a donkey', impossible.

Note that this uses names for propositions (e.g., A).

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Note that this uses names for propositions (e.g., A). We can distinguish:

- “You know that A is true”: in every epistemically possible world, whatever A happens to name in that world is true.
- “Of A, you know that it is true: whatever A actually names in the current world, that thing is true in every epistemically possible world.”



There is *so much more*

- If you would like a list of references on some specific topic, let me know your level of comfort with Latin, and I can send you a bibliography of relevant primary and secondary literature.
- I also have an ever growing list of specific topics I am interested in working on, more than I can ever tackle myself. I'm happy to share research ideas, and I love to collaborate.

Contact me at:

uckelman@uni-heidelberg.de