The knowability paradox. An approach in QML

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Lund 22-24 Mai 2014

Outline

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1. A paradox for verificationism?
2. the reformulation strategy and its problems
3. A toy example and a possible way out
4. The problem in QML
5. Conclusions

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The problem

Acceptable premises (for a verificationist)

(VT) Every true proposition is *possibly known*

(NO) There are unknown truths.

. . . and an undesirable conclusion

(O) All truths are known.

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Fitch 1963 (A modal collapse)

1	$\forall\phi\ (\phi \rightarrow \Diamond K\phi)$	[Ass.]
2	$\exists\psi\ (\psi \wedge \neg K\psi)$	[Ass.]
3	$(\psi_1 \wedge \neg K\psi_1)$	[instantiation of 2]
4	$(\psi_1 \wedge \neg K\psi_1) \rightarrow \Diamond K(\psi_1 \wedge \neg K\psi_1)$	[1,3,substitution]
5	$\Diamond K(\psi_1 \wedge \neg K\psi_1) \rightarrow \Diamond(K\psi_1 \wedge K\neg K\psi_1)$	[K-distribution]
6	$\Diamond(K\psi_1 \wedge K\neg K\psi_1) \rightarrow \Diamond(K\psi_1 \wedge \neg K\psi_1)$	[axiom T]
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Different solutions...

1. Weakening strategies (intuitionistic - paraconsistent solutions)
2. Restriction strategies (limiting the substitution instances of VT)
3. Reformulation strategies (rewriting VT)

... for different problems

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Reformulation (Edgington 1985, S.& R., Kvanvig)

$$\Diamond + K \neq \text{knowability}$$

- Knowability concerns "our" world
- $\Diamond K$ sends us somewhere else

Edgington: VT is better expressed with use of some rigidifying operators such as @ (actually)

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1. Actual truths are necessary.
   Not only: every concatenation of operators collapses.

$$\Box @\phi \leftrightarrow @\phi$$

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Some considerations

- Both VT and NO quantify over actual and possible agents and situations.
- Propositional modal logic is a simple and useful tool, but maybe opaque.
- A quantified modal language is a finer-grained tool and helps clarifying "metaphysical" choices from the beginning (e.g. actualist quantification vs. possibilist quantification)

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Ambiguity of VT and NO

Two different meanings of VT

$$\forall\Phi(\Phi \rightarrow \exists x \Diamond K_x \Phi)$$

or

$$\forall\Phi(\Phi \rightarrow \Diamond \exists x K_x \Phi)$$

and of NO

$$\exists\Phi(\Phi \wedge \forall x \neg K_x \Phi)$$

or

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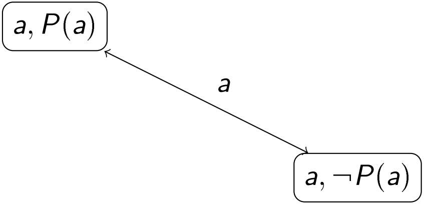
where is the problem

- A truth is a set of possible worlds (denoted $[\![\phi]\!]$)
- For a given w, $w \in [\![K\phi]\!]$ iff $R[w] \subseteq [\![\phi]\!]$
- If $\phi = p \wedge \neg Kp$ then $w \in [\![K\phi]\!]$ only if $R[w] \subseteq [\![p \wedge \neg Kp]\!]$.
- Impossible
- $[\![K\phi]\!]$ is empty.

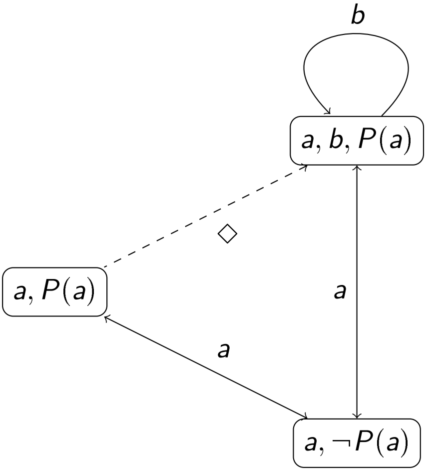
Toy example: a most peculiar man

$a, P(a)$

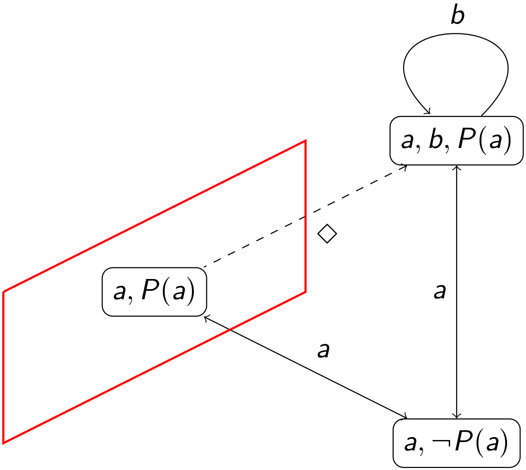
. . . who doesn't know



and a real nowhere man



who doesn't know "P(a) and nobody knows that P(a)"



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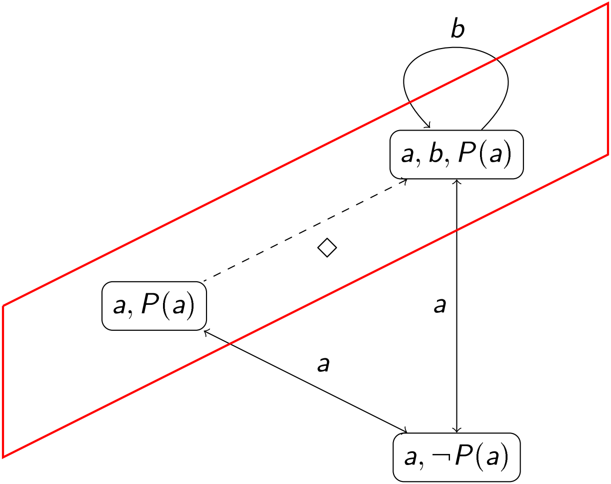

Indexicality (see also Kvanvig 1995)

Indexicality is hidden everywhere

$$P(\ a) \wedge \neg\ \exists x K_{\ x} P(\ a)$$

we are usually able to determine it

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...but maybe not always in a modal context

$$\Diamond \exists y K_y (P(?a) \wedge \neg ? \exists x K_{?x} P(?a))$$

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Language

The language contains nominals i, j, l, state variables u, v, z for possible worlds, rigidifying operators $@_u$, $@_i$ and a downarrow binders $\downarrow u$.

Terms

$$x \mid c \mid u : t \mid i : t \mid f(t_1, \ldots, t_n)$$

Formulas

$$R(t_1, \ldots, t_n) \mid t_1 = t_2 \mid u \mid i \mid \neg\phi \mid (\phi \vee \psi) \mid (\exists x)\phi \mid \Diamond\phi \mid K_t\phi \mid @_u\phi \mid @_i\phi \mid \downarrow v.\phi$$

Semantics

Definition (Models)

$\mathcal{M} = \langle W, \mathcal{R}_\diamond, \mathcal{R}_{d\in\mathbb{D}}, \mathbb{D}, \mathcal{D}, I\rangle$ is a varying domain frame + an interpretation of a language

Definition (Valuation)

ν is a valuation in the model $\mathcal{M} = \langle W, \mathcal{R}_\diamond, \mathcal{R}_{d\in\mathbb{D}}, \mathbb{D}, \mathcal{D}, \mathcal{I}\rangle$ if ν is a function ν : **VAR** $\rightarrow \mathbb{W} \cup \mathbb{D}$

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$$
\begin{array}{rcl}
\mathcal{M}, w \models_\nu R(t_1, \ldots, t_n) & \text{iff} & ((t_1)_w^{\mathcal{M},\nu}, \ldots, (t_n)_w^{\mathcal{M},\nu}) \in R_w^{\mathcal{M},\nu} \\
\mathcal{M}, w \models_\nu t_1 = t_2 & \text{iff} & (t_1)_w^{\mathcal{M},\nu} = (t_n)_w^{\mathcal{M},\nu} \\
\mathcal{M}, w \models_\nu i & \text{iff} & \mathcal{I}(i) = w \\
\mathcal{M}, w \models_\nu v & \text{iff} & \nu(v) = w \\
\mathcal{M}, w \models_\nu \neg\phi & \text{iff} & \mathcal{M}, w \not\models_\nu \phi \\
\mathcal{M}, w \models_\nu \phi \vee \psi & \text{iff} & \mathcal{M}, w \models_\nu \phi \text{ or } \mathcal{M}, w \models_\nu \psi \\
\mathcal{M}, w \models_\nu \diamond\phi & \text{iff} & \text{there is a } w' \text{ s.t. } wRw' \text{ and } \mathcal{M}, v \models_\nu \phi \\
\mathcal{M}, w \models_\nu K_t\phi & \text{iff} & \text{for all } w' \text{ s.t. } wR_{(t)_w^{\mathcal{M},\nu}} w' \text{ and } \mathcal{M}, w' \models_\nu \phi \\
\mathcal{M}, w \models_\nu \exists x\phi & \text{iff} & \text{for some } \nu' \overset{x}{\sim} \nu \text{ in } w \text{ s.t. } \mathcal{M}, w \models_{\nu'} \phi \\
\mathcal{M}, w \models_\nu @_i\phi & \text{iff} & \mathcal{M}, \mathcal{I}(i) \models_\nu \phi \\
\mathcal{M}, w \models_\nu @_v\phi & \text{iff} & \mathcal{M}, \nu(v) \models_\nu \phi \\
\mathcal{M}, w \models_\nu \downarrow v.\phi & \text{iff} & \text{for some } \nu' \overset{v}{\sim} \nu \text{ s.t. } \nu'(v) = w \text{ and } \mathcal{M}, w \models_{\nu'} \phi
\end{array}
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Knowability: two options

knowability of $P(a) \wedge \neg \exists x K_x P(a)$

$$\downarrow v \Diamond \exists y K_y @_v (P(a) \wedge \neg \exists z K_z P(a)) \quad (1)$$

$$\downarrow v \Diamond \downarrow u \exists y K_y (P(v:a) \wedge \neg @_v \exists z @_u K_{v:z} P(v:a)) \quad (2)$$

(1) is Edgington sense of knowability, while (2) is not.

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where:

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Moral

1. (VT) is no more a substitution free schema but is the outcome of a systematic translation: "If ϕ then it is possibly known that ϕ... as originally meant
2. rigidification in ϕ is preserved by σ^v_u, e.g. $P(a) \wedge \neg\exists x K_x \downarrow z.P(z : a)$ gives $\downarrow v \Diamond \downarrow u \exists y K_y (P(v : a) \wedge \neg @_v \exists z @_u K_{v:x} \downarrow z.P(v : z : a))$
3. Paradoxality is avoided in a way that prevents usual problems of reformulation strategies
4. But σ^v_u is just one possible way of specifying hidden indexicality
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Thank you!