A fuzzy hybrid logic and opinion dynamics in social networks

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Modality and Modalities
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Social networks
Social networks

Facebook and Twitter
Social networks

Facebook and Twitter

Physical interaction
Social networks

Facebook and Twitter

Physical interaction

Co-authorship
Social networks

Dynamic processes in social networks

Facebook and Twitter
Physical interaction
Co-authorship
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Diffusion of innovation

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Diffusion of innovation
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Facebook and Twitter
Epidemics

Dynamic processes in social networks
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Opinion dynamics
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Dynamic processes in social networks
Modal logic and social networks

- Social networks are (directed or undirected) graphs
- Modal and hybrid logic (in an egocentric reading) to describe network structures
- The dynamics are often local dynamics
- A dynamic modal logic where preconditions are described by modal formulas
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Opinion dynamics in social networks

Examples
- Deliberation in groups, such as juries or boards of directors
- Public opinion on matters such as global warming or surveillance of private citizen
- Twitter storms

Simple models of opinion dynamics
- Morris DeGroot
- Keith Lehrer and Carl Wagner
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Outline

- DeGroot and Lehrer’s model of opinion dynamics
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research
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DeGroot and Lehrer’s model of opinion dynamics

- A model independently developed by DeGroot (1974) and Lehrer (1976)
- A group of $k$ individuals
- Each with an opinion $O_i \in [0, 1]$
- The question is how to aggregate these opinions into a joint opinion – a group consensus?
- One solution, a weighted average: $\sum_{i=1}^{k} w_i O_i$
- The question then is how to choose the weights $w_i$?
- Through a process of deliberation/communication
- Initially each $i$ assigns a weight $w_{ij}$ to the opinion of each $j$ (s.t. $\sum_{j=1}^{k} w_{ij} = 1$) – a kind of trust
- Then, each $i$ update her opinion as an weighted average (using the $w'_{ij}$s): $O_i(\text{new}) = \sum_{j=1}^{k} w_{ij} O_j$. 
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A fuzzy hybrid logic and opinion dynamics in social networks
DeGroot and Lehrer’s model of opinion dynamics

- This updating can proceed until a consensus is reached.
- Let $O_{i,n}$ be the opinion of $i$ at time step $n$.

Then, the general updating rule is: $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n-1}$

A consensus is reached if for some $n$:

$O_{i,n} = O_{i,n+1} = O_{j,n} = O_{j,n+1}$ for all $i$ and $j$, or

$\lim_{n \to \infty} O_{i,n} = \lim_{n \to \infty} O_{j,n}$ for all $i$ and $j$

The update rule in matrix notation:

$O_n = WO_{n-1} = W^n O_0$

where $O$ is the $k \times 1$ column vector of the $O'_{i,n}$s and $W$ is the $k \times k$ matrix with entries $(w_{ij})$

This is essentially a Markov chain

Using results from Markov chain theory, DeGroot showed that consensus is reached iff the directed network given by the links $w_{ij}$ with $w_{ij} > 0$, is strongly connected and aperiodic.
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A fuzzy hybrid logic

The basic idea

- The initial state of the DeGroot model is a weighted graph
- There is one propositional variable $O$ that is assigned a value in $[0, 1]$ at each agent in the network ("in each possible world")
- The "accessibility relation" is many-valued from $[0, 1]$ as well
- This naturally leads to a many-valued modal logic in the line of Fitting (1992a,b)
- Taking "+" as join and "·" as meet, the averaging at each step $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n}$ is join of meets – the semantics of the ◊ modality of Fitting (1992a,b)
- However, this doesn’t quite work...
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A fuzzy hybrid logic

Definition (Syntax)

The static language $\mathcal{L}_S$ is defined in the following way:

$$\varphi ::= P \mid i \mid (\varphi \leq q) \mid (\varphi \geq q) \mid \neg \varphi \mid (\varphi \land \varphi) \mid (\varphi \rightarrow \varphi) \mid \diamond \varphi \mid \circ_i \varphi \mid E \varphi \mid [P := \varphi] \varphi \mid [P := \varphi]^* \varphi,$$

where $p \in \text{PROP}$ (prop. var.), $i \in \text{NOM}$ (nominal), and $q \in [0, 1] \cap \mathbb{Q}$. 
A fuzzy hybrid logic and opinion dynamics in social networks

Definition (Network model)

A network model is a tuple $\mathcal{M} = \langle A, R, g, V \rangle$, where $A$ is a non-empty set of individuals/agents, $R : A \times A \rightarrow [0, 1]$ a trust distribution such that for all $a \in A$, $\sum_{b \in A} R(a, b) = 1$, $g : \text{NOM} \rightarrow A$ is a naming function assigning agents to each nominal, and $V : A \times \text{PROP} \rightarrow [0, 1]$ is a valuation assigning truth values for each agent to each propositional variable.
Definition (Semantics)

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\bar{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

- $\bar{V}(a, P) = V(a, P)$
- $\bar{V}(a, i) = \begin{cases} 1 & \text{if } a = g(i) \\ 0 & \text{otherwise} \end{cases}$
- $\bar{V}(a, \varphi \leq q) = \begin{cases} 1 & \text{if } \bar{V}(a, \varphi) \leq q \\ 0 & \text{otherwise} \end{cases}$
- $\bar{V}(a, \neg \varphi) = 1 - \bar{V}(a, \varphi)$
- $\bar{V}(a, \varphi \land \psi) = \max\{0, \bar{V}(a, \varphi) + \bar{V}(a, \psi) - 1\}$
- $\bar{V}(a, \varphi \rightarrow \psi) = \min\{1, 1 - \bar{V}(a, \varphi) + \bar{V}(a, \psi)\}$
- $\bar{V}(a, \Diamond \varphi) = \sum_{b \in A} R(a, b) \bar{V}(b, \varphi)$
- $\bar{V}(a, \Diamond_i \varphi) = \bar{V}(g(i), \varphi)$
- $\bar{V}(a, E \varphi) = \sup\{\bar{V}(b, \varphi) \mid b \in A\}$
A fuzzy hybrid logic

Definition (Semantics)

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\tilde{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

\[
\begin{align*}
\tilde{V}(a, P) &= V(a, P) \\
\tilde{V}(a, i) &= \begin{cases} 1 & \text{if } a = g(i) \\ 0 & \text{otherwise} \end{cases} \\
\tilde{V}(a, \varphi \leq q) &= \begin{cases} 1 & \text{if } \tilde{V}(a, \varphi) \leq q \\ 0 & \text{otherwise} \end{cases} \\
\tilde{V}(a, \neg \varphi) &= 1 - \tilde{V}(a, \varphi) \\
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\tilde{V}(a, \Diamond \varphi) &= \sum_{b \in A} R(a, b) \tilde{V}(b, \varphi) \\
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Definition (Semantics (continued))

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\tilde{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

$$\tilde{V}(a, [P := \varphi]\psi) = \tilde{V}_{P:=\varphi}(a, \psi)$$

$$\tilde{V}(a, [P := \Diamond \varphi]^*\psi) = \sup\{\tilde{V}(a, [P := \Diamond \varphi]^n\psi) \mid n \in \mathbb{N}\}$$

Definition (Updated network models)

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \text{PROP}$, and a $\varphi \in \mathcal{L}_D$, we let the updated model $\mathcal{M}_{P:=\varphi}$ be $\langle A, R, g, V_{P:=\varphi} \rangle$, where

$$V_{P:=\varphi}(a, Q) = V(a, Q), \quad \text{for all } Q \in \text{PROP}\backslash\{P\} \text{ and all } a \in A$$

$$V_{P:=\varphi}(a, P) = \tilde{V}(a, \varphi), \quad \text{for all } a \in A$$

Definition (Validity)

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \text{PROP}$, and a formula $\varphi$, $\varphi$ is valid in $\mathcal{M}$ iff $V(a, \varphi) = 1$ for all $a \in A$. 
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\tilde{V}(a, [P := \varphi] \psi) = \tilde{V}_{P := \varphi}(a, \psi)
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A fuzzy hybrid logic and opinion dynamics in social networks
Properties of the logic

- $\bar{V}(a, \neg\Diamond\neg\varphi) = \bar{V}(a, \Diamond\varphi)$

- $\bar{V}(a, U\varphi) = \inf\{\bar{V}(b, \varphi) \mid b \in A\}$ \hspace{1cm} (\(U\varphi := \neg\neg\varphi\))

- $\bar{V}(a, \varphi \rightarrow \psi) = 1$ \text{ iff } $\bar{V}(a, \varphi) \leq \bar{V}(a, \psi)$

- $\bar{V}(a, \varphi \leq q) = 1$ \text{ iff } $\bar{V}(a, \varphi) \leq q$

- $\bar{V}(a, \varphi = q) = 1$ \text{ iff } $\bar{V}(a, \varphi) = q$

- $\bar{V}(a, @iP) = V(g(i), P)$

- $\bar{V}(a, @i\Diamond j) = R(g(i), g(j))$
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DeGroot and Lehrer’s model of opinion dynamics

A fuzzy hybrid logic

Reasoning about opinion dynamics in social networks

Concluding remarks and future research
Formalizing DeGroot’s model

- Let $A$ be the set of individuals making up the group
- Let $P$ be the proposition that the agents have opinions about (i.e. $PROP = \{P\}$)
- The initial opinion of agent $a$ towards $P$ is $V(a, P)$
- $R(a, b)$ is $w_{ab}$, i.e. the weight that agent $a$ puts on the opinion of agent $b$
- Choose any function $g : NOM \rightarrow A$ and set $NOM$

DeGroot’s updating mechanism corresponds to a substitution of $P$ by $\Diamond P$.

- I.e. an application of the modality $[P := \Diamond P]$
- $\varphi$ is true after 7 steps of the DeGroot dynamics: $[P := \Diamond P]^7 \varphi$
- That the agents is a consensus of assigning $q$ to $P$: $(UP) = q$
- The agents are in a consensus: $EP \rightarrow UP$
Formalizing DeGroot’s model

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  - The agents are in a consensus: $EP \rightarrow UP$
Formalizing DeGroot’s model

- The agents are in a consensus: $EP \rightarrow UP$
- If consensus is reached after a finite number of steps $n$, then
  $\bar{V}(a, [P := \Diamond P]^*(EP \rightarrow UP)) = 1$.
- However, the other doesn’t quite work
- Moreover, it does not guarantees convergence to a consensus

Theorem

Given a network model $M$. Then, a consensus is reached in $M$ under the DeGroot dynamics (i.e. successive updates with $[P := \Diamond P]$), if and only if, the formula

$[P := \Diamond P]^*(P := \Diamond P)^*(EP \rightarrow UP)$

is valid in $M$.

The proof uses that: $\bar{V}(a, [P := \Diamond P]^*(P := \Diamond P)^*(EP \rightarrow UP)) = \lim \inf_{n \to \infty} \bar{V}(a, [P := \Diamond P]^n(EP \rightarrow UP))$

- properties in the limit can be expressed as: $[P := \Diamond P]^*(P := \Diamond P)^* \varphi$
Formalizing DeGroot’s model

- The agents are in a consensus: $EP \rightarrow UP$
- If consensus is reached after a finite number of steps $n$, then
  $\bar{V}(a, [P := \Diamond P]^*(EP \rightarrow UP)) = 1$.
- However, the other doesn’t quite work
- Moreover, it does not guarantees convergence to a consensus

Theorem

Given a network model $M$. Then, a consensus is reached in $M$ under the DeGroot dynamics (i.e. successive updates with $[P := \Diamond P]$), if and only if, the formula

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**Theorem**

*Given a network model $\mathcal{M}$. Then, a consensus is reached in $\mathcal{M}$ under the DeGroot dynamics (i.e. successive updates with $[P := \Diamond P]$), if and only if, the formula $[P := \Diamond P]*\langle P := \Diamond P\rangle* (EP \rightarrow UP)$ is valid in $\mathcal{M}$.*

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- properties in the limit can be expressed as: \( [P := \Diamond P]^*\langle P := \Diamond P \rangle^* \phi \)
Outline

- DeGroot and Lehrer’s model of opinion dynamics
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research
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Conclusions

- Modal logic is suited for reasoning about simple social network dynamics
- More complex dynamics such as DeGroot's can also be captured in proper extensions of modal logic

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LOGICAL METHODS FOR SOCIAL NETWORK ANALYSIS

Future research

- Develop a proof theory or decision procedure for the logic
- Compare to probabilistic logics
- Compare to logics for Markov chains etc.
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