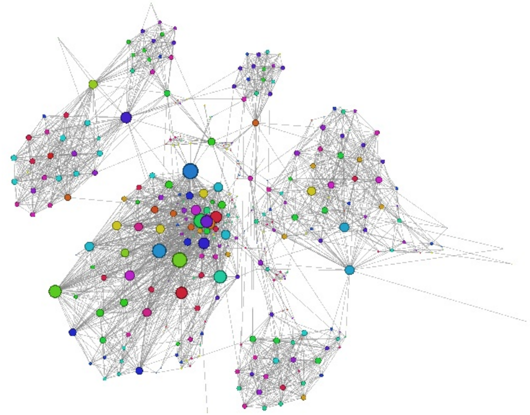
# A fuzzy hybrid logic and opinion dynamics in social networks

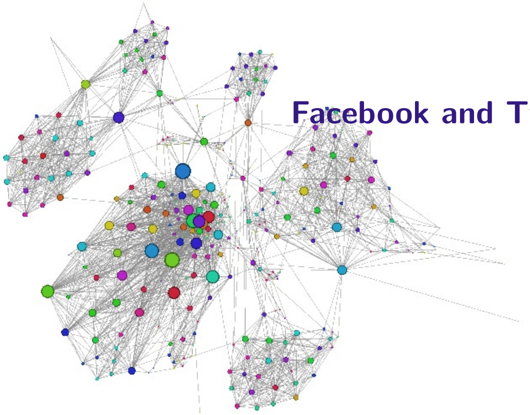
**Jens Ulrik Hansen**
Department of Philosophy,
Lund University, Sweden

Modality and Modalities
May 22–24, 2014, Lund

# Social networks

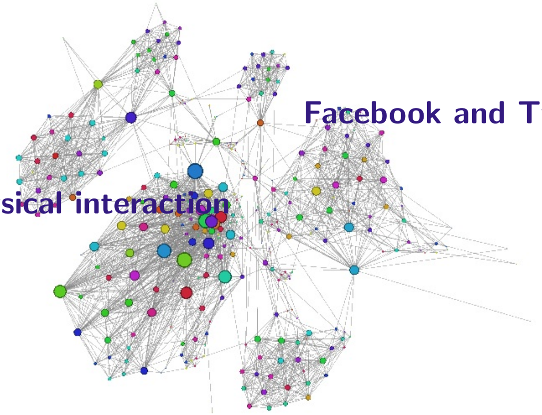


# Social networks

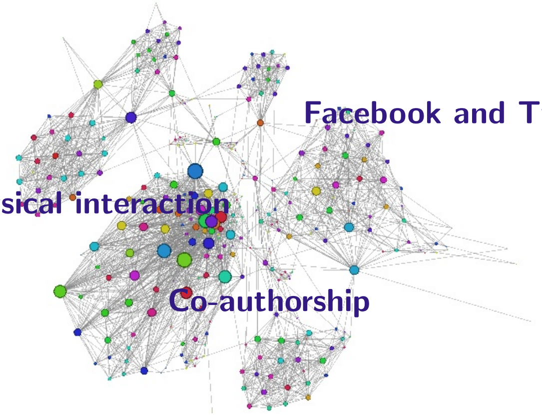


Facebook and Twitter

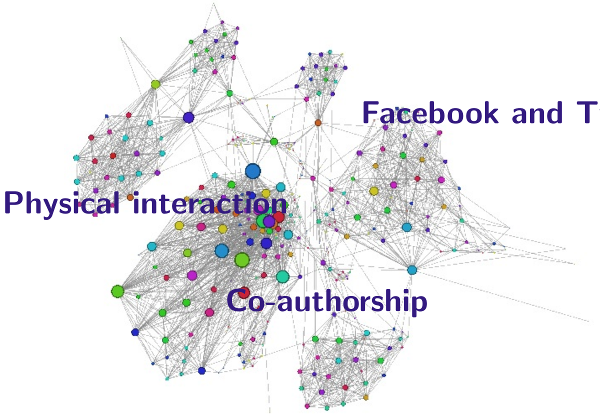
# Social networks



# Social networks

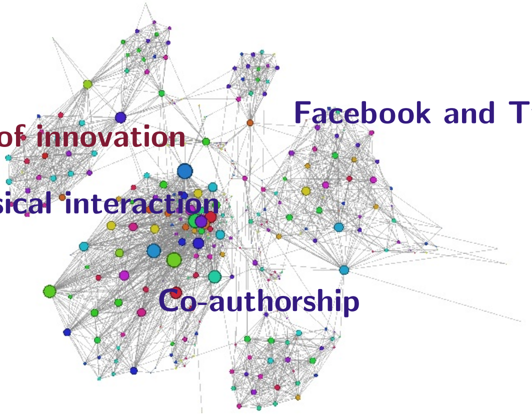


# Social networks



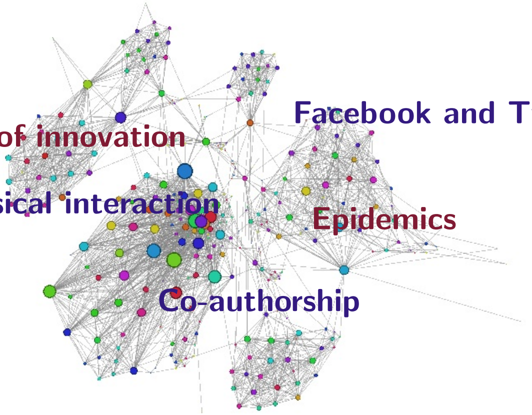

## Dynamic processes in social networks

# Social networks



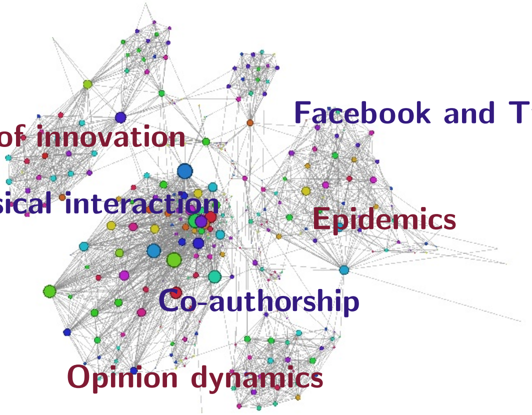

**Dynamic processes in social networks**

# Social networks




**Dynamic processes in social networks**

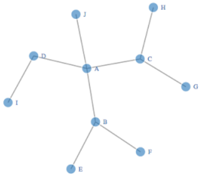
# Social networks




**Dynamic processes in social networks**

# Modal logic and social networks

- Social networks are (directed or undirected) graphs
- Modal and hybrid logic (in an egocentric reading) to describe network structures
- The dynamics are often local dynamics
- A dynamic modal logic where preconditions are described by modal formulas



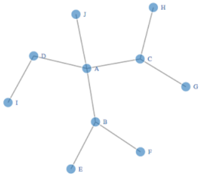
# Modal logic and social networks

– Social networks are (directed or undirected) graphs

– Modal and hybrid logic (in an egocentric reading) to describe network structures

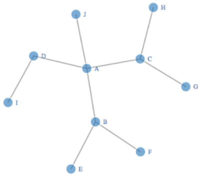
– The dynamics are often local dynamics

– A dynamic modal logic where preconditions are described by modal formulas



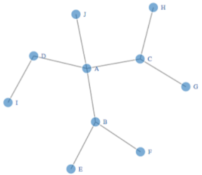
# Modal logic and social networks

- Social networks are (directed or undirected) graphs
- Modal and hybrid logic (in an egocentric reading) to describe network structures
- The dynamics are often local dynamics
- A dynamic modal logic where preconditions are described by modal formulas



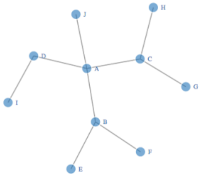
# Modal logic and social networks

- Social networks are (directed or undirected) graphs
- Modal and hybrid logic (in an egocentric reading) to describe network structures
- The dynamics are often local dynamics
- A dynamic modal logic where preconditions are described by modal formulas



# Modal logic and social networks

- Social networks are (directed or undirected) graphs
- Modal and hybrid logic (in an egocentric reading) to describe network structures
- The dynamics are often local dynamics
- A dynamic modal logic where preconditions are described by modal formulas



# Opinion dynamics in social networks

## Examples

- Deliberation in groups, such as juries or broads of directors
- public opinion on matters such as global warming or surveillance of private citizen
- Twitter storms

## Simple models of opinion dynamics

- Morris DeGroot
- Keith Lehrer and Carl Wagner

# Opinion dynamics in social networks

## Examples

- Deliberation in groups, such as juries or broads of directors
- public opinion on matters such as global warming or surveillance of private citizen
- Twitter storms

## Simple models of opinion dynamics

- Morris DeGroot
- Keith Lehrer and Carl Wagner

# Opinion dynamics in social networks

## Examples

- Deliberation in groups, such as juries or broads of directors
- public opinion on matters such as global warming or surveillance of private citizen
- Twitter storms

## Simple models of opinion dynamics

- Morris DeGroot
- Keith Lehrer and Carl Wagner

# Outline

- DeGroot and Lehrer's model of opinion dynamics
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research

# Outline

- **DeGroot and Lehrer's model of opinion dynamics**
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research

# DeGroot and Lehrer's model of opinion dynamics

- A model independently developed by DeGroot (1974) and Lehrer (1976)
- A group of $k$ individuals
- Each with an opinion $O_i \in [0, 1]$
- The question is how to aggregate these opinions into a joint opinion – a group consensus?
- One solution, a weighted average: $\sum_{i=1}^{k} w_i O_i$
- The question then is how to choose the weights $w_i$?
- Through a process of deliberation/communication
- Initially each $i$ assigns a weight $w_{ij}$ to the opinion of each $j$ (s.t. $\sum_{j=1}^{k} w_{ij} = 1$) – a kind of trust
- Then, each $i$ update her opinion as an weighted average (using the $w_{ij}'s$): $O_i(new) = \sum_{j=1}^{k} w_{ij} O_j$.

# DeGroot and Lehrer's model of opinion dynamics

- A model independently developed by DeGroot (1974) and Lehrer (1976)
- A group of $k$ individuals
- Each with an opinion $O_i \in [0, 1]$
- The question is how to aggregate these opinions into a joint opinion – a group consensus?
- One solution, a weighted average: $\sum_{i=1}^{k} w_i O_i$
- The question then is how to choose the weights $w_i$?
- Through a process of deliberation/communication
- Initially each $i$ assigns a weight $w_{ij}$ to the opinion of each $j$ (s.t. $\sum_{j=1}^{k} w_{ij} = 1$) – a kind of trust
- Then, each $i$ update her opinion as an weighted average (using the $w_{ij}'s$): $O_i(new) = \sum_{j=1}^{k} w_{ij} O_j$.

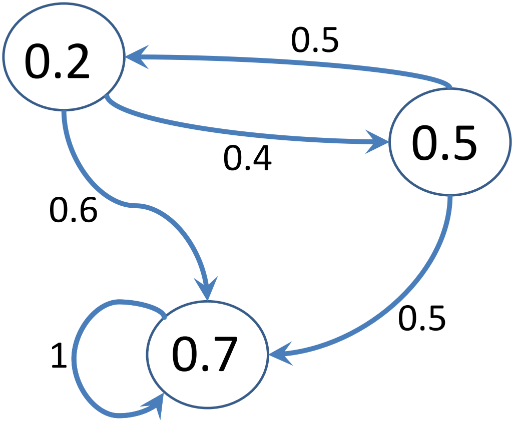
# DeGroot and Lehrer's model of opinion dynamics

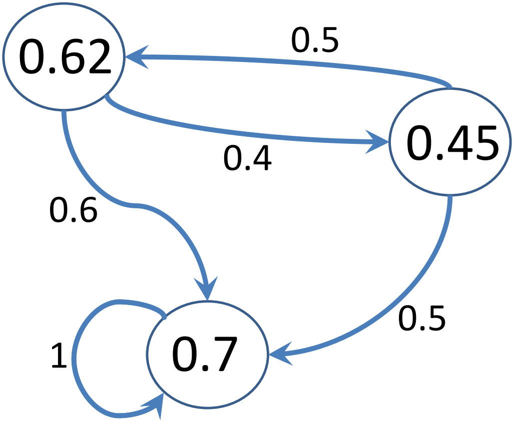
- A model independently developed by DeGroot (1974) and Lehrer (1976)
- A group of $k$ individuals
- Each with an opinion $O_i \in [0, 1]$
- The question is how to aggregate these opinions into a joint opinion – a group consensus?
- One solution, a weighted average: $\sum_{i=1}^{k} w_i O_i$
- The question then is how to choose the weights $w_i$?
- Through a process of deliberation/communication
- Initially each $i$ assigns a weight $w_{ij}$ to the opinion of each $j$ (s.t. $\sum_{j=1}^{k} w_{ij} = 1$) – a kind of trust
- Then, each $i$ update her opinion as an weighted average (using the $w_{ij}'s$): $O_i(new) = \sum_{j=1}^{k} w_{ij} O_j$.

# DeGroot and Lehrer's model of opinion dynamics

- A model independently developed by DeGroot (1974) and Lehrer (1976)
- A group of $k$ individuals
- Each with an opinion $O_i \in [0,1]$
- The question is how to aggregate these opinions into a joint opinion – a group consensus?
- One solution, a weighted average: $\sum_{i=1}^{k} w_i O_i$
- The question then is how to choose the weights $w_i$?
- Through a process of deliberation/communication
- Initially each $i$ assigns a weight $w_{ij}$ to the opinion of each $j$ (s.t. $\sum_{j=1}^{k} w_{ij} = 1$) – a kind of trust
- Then, each $i$ update her opinion as an weighted average (using the $w_{ij}'s$): $O_i(new) = \sum_{j=1}^{k} w_{ij} O_j$.

# DeGroot and Lehrer's model of opinion dynamics




0.62
0.5
0.4
0.45
0.6
0.5
1
0.7

# DeGroot and Lehrer's model of opinion dynamics

- This updating can proceed until a consensus is reached.
- Let $O_{i,n}$ be the opinion of $i$ at time step $n$.
- Then, the general updating rule is: $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n-1}$
- A consensus is reached if for some $n$:
  $O_{i,n} = O_{i,n+1} = O_{j,n} = O_{j,n+1}$ for all $i$ and $j$, or
  $\lim_{n\to\infty} O_{i,n} = \lim_{n\to\infty} O_{j,n}$ for all $i$ and $j$
- The update rule in matrix notation: $\mathbf{O}_n = \mathbf{W}\mathbf{O}_{n-1} = \mathbf{W}^n\mathbf{O}_0$
- where $\mathbf{O}$ is the $k \times 1$ column vector of the $O'_{i,n}s$ and $\mathbf{W}$ is the $k \times k$ matrix with entries $(w_{ij})$
- This is essential a Markov chain
- Using results from Markov chain theory, DeGroot showed that *consensus is reached iff the directed network given by the links $w_{ij}$ with $w_{ij} > 0$, is strongly connected and aperiodic.*

# DeGroot and Lehrer's model of opinion dynamics

- This updating can proceed until a consensus is reached.
- Let $O_{i,n}$ be the opinion of $i$ at time step $n$.
- Then, the general updating rule is: $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n-1}$
- A consensus is reached if for some $n$: $O_{i,n} = O_{i,n+1} = O_{j,n} = O_{j,n+1}$ for all $i$ and $j$, or $\lim_{n\to\infty} O_{i,n} = \lim_{n\to\infty} O_{j,n}$ for all $i$ and $j$
- The update rule in matrix notation: $\mathbf{O}_n = \mathbf{W}\mathbf{O}_{n-1} = \mathbf{W}^n \mathbf{O}_0$
- where $\mathbf{O}$ is the $k \times 1$ column vector of the $O'_{i,n}s$ and $\mathbf{W}$ is the $k \times k$ matrix with entries $(w_{ij})$
- This is essential a Markov chain
- Using results from Markov chain theory, DeGroot showed that *consensus is reached iff the directed network given by the links $w_{ij}$ with $w_{ij} > 0$, is strongly connected and aperiodic.*

# DeGroot and Lehrer's model of opinion dynamics

- This updating can proceed until a consensus is reached.
- Let $O_{i,n}$ be the opinion of $i$ at time step $n$.
- Then, the general updating rule is: $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n-1}$
- A consensus is reached if for some $n$:
  $O_{i,n} = O_{i,n+1} = O_{j,n} = O_{j,n+1}$ for all $i$ and $j$, or
  $\lim_{n\to\infty} O_{i,n} = \lim_{n\to\infty} O_{j,n}$ for all $i$ and $j$
- The update rule in matrix notation: $\mathbf{O}_n = \mathbf{W}\mathbf{O}_{n-1} = \mathbf{W}^n\mathbf{O}_0$
- where $\mathbf{O}$ is the $k \times 1$ column vector of the $O'_{i,n}s$ and $\mathbf{W}$ is the $k \times k$ matrix with entries $(w_{ij})$
- This is essential a Markov chain
- Using results from Markov chain theory, DeGroot showed that *consensus is reached iff the directed network given by the links $w_{ij}$ with $w_{ij} > 0$, is strongly connected and aperiodic.*

# DeGroot and Lehrer's model of opinion dynamics

- This updating can proceed until a consensus is reached.
- Let $O_{i,n}$ be the opinion of $i$ at time step $n$.
- Then, the general updating rule is: $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n-1}$
- A consensus is reached if for some $n$:
  $O_{i,n} = O_{i,n+1} = O_{j,n} = O_{j,n+1}$ for all $i$ and $j$, or
  $\lim_{n\to\infty} O_{i,n} = \lim_{n\to\infty} O_{j,n}$ for all $i$ and $j$
- The update rule in matrix notation: $\mathbf{O}_n = \mathbf{W}\mathbf{O}_{n-1} = \mathbf{W}^n\mathbf{O}_0$
- where $\mathbf{O}$ is the $k \times 1$ column vector of the $O'_{i,n}s$ and $\mathbf{W}$ is the $k \times k$ matrix with entries $(w_{ij})$
- This is essential a Markov chain
- Using results from Markov chain theory, DeGroot showed that *consensus is reached iff the directed network given by the links $w_{ij}$ with $w_{ij} > 0$, is strongly connected and aperiodic.*

# DeGroot and Lehrer's model of opinion dynamics

- This updating can proceed until a consensus is reached.
- Let $O_{i,n}$ be the opinion of $i$ at time step $n$.
- Then, the general updating rule is: $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n-1}$
- A consensus is reached if for some $n$:
  $O_{i,n} = O_{i,n+1} = O_{j,n} = O_{j,n+1}$ for all $i$ and $j$, or
  $\lim_{n\to\infty} O_{i,n} = \lim_{n\to\infty} O_{j,n}$ for all $i$ and $j$
- The update rule in matrix notation: $\mathbf{O}_n = \mathbf{W}\mathbf{O}_{n-1} = \mathbf{W}^n\mathbf{O}_0$
- where $\mathbf{O}$ is the $k \times 1$ column vector of the $O'_{i,n}s$ and $\mathbf{W}$ is the $k \times k$ matrix with entries $(w_{ij})$
- This is essential a Markov chain
- Using results from Markov chain theory, DeGroot showed that *consensus is reached iff the directed network given by the links $w_{ij}$ with $w_{ij} > 0$, is strongly connected and aperiodic.*

# Outline

- DeGroot and Lehrer's model of opinion dynamics
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research

# Outline

- DeGroot and Lehrer's model of opinion dynamics
- **A fuzzy hybrid logic**
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research

**The basic idea**

- The initial state of the DeGroot model is a weighted graph
- There is one propositional variable $O$ that is assigned a value in $[0,1]$ at each agent in the network ("in each possible world")
- The "accessibility relation" is many-valued from $[0,1]$ as well
- This naturally leads to a many-valued modal logic in the line of Fitting (1992a,b)
- Taking "$+$" as join and "$\cdot$" as meet, the averaging at each step $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n}$ is join of meets – the semantics of the $\Diamond$ modality of Fitting (1992a,b)
- However, this doesn't quite work...

**The basic idea**

- The initial state of the DeGroot model is a weighted graph
- There is one propositional variable $O$ that is assigned a value in $[0, 1]$ at each agent in the network ("in each possible world")
- The "accessibility relation" is many-valued from $[0, 1]$ as well
- This naturally leads to a many-valued modal logic in the line of Fitting (1992a,b)
- Taking "+" as join and "·" as meet, the averaging at each step $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n}$ is join of meets – the semantics of the $\Diamond$ modality of Fitting (1992a,b)
- However, this doesn't quite work...

**The basic idea**

- The initial state of the DeGroot model is a weighted graph
- There is one propositional variable $O$ that is assigned a value in $[0, 1]$ at each agent in the network ("in each possible world")
- The "accessibility relation" is many-valued from $[0, 1]$ as well
- This naturally leads to a many-valued modal logic in the line of Fitting (1992a,b)
- Taking "$+$" as join and "$\cdot$" as meet, the averaging at each step $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n}$ is join of meets – the semantics of the $\Diamond$ modality of Fitting (1992a,b)
- However, this doesn't quite work...

**The basic idea**

- The initial state of the DeGroot model is a weighted graph
- There is one propositional variable $O$ that is assigned a value in $[0, 1]$ at each agent in the network ("in each possible world")
- The "accessibility relation" is many-valued from $[0, 1]$ as well
- This naturally leads to a many-valued modal logic in the line of Fitting (1992a,b)
- Taking "+" as join and "·" as meet, the averaging at each step $O_{i,n} = \sum_{j=1}^{k} w_{ij} O_{j,n}$ is join of meets – the semantics of the $\Diamond$ modality of Fitting (1992a,b)
- However, this doesn't quite work...

# A fuzzy hybrid logic

**Definition (Syntax)**

The static language $\mathcal{L}_S$ is defined in the following way:

$$\varphi \quad ::= \quad P \mid i \mid (\varphi \leq q) \mid (\varphi \geq q) \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \rightarrow \varphi) \mid \Diamond\varphi \mid @_i\varphi \mid E\varphi \mid [P := \varphi]\varphi \mid [P := \varphi]^*\varphi \ ,$$

where $p \in$ PROP (prop. var.), $i \in$ NOM (nominal), and $q \in [0, 1] \cap \mathbb{Q}$.

# A fuzzy hybrid logic

**Definition (Network model)**

A *network model* is a tuple $\mathcal{M} = \langle A, R, g, V \rangle$, where $A$ is a non-empty set of *individuals/agents*, $R : A \times A \to [0, 1]$ a *trust distribution* such that for all $a \in A$, $\sum_{b \in A} R(a, b) = 1$, $g : \text{NOM} \to A$ is a *naming function* assigning agents to each nominal, and $V : A \times \text{PROP} \to [0, 1]$ is a *valuation* assigning truth values for each agent to each propositional variable.

# A fuzzy hybrid logic

## Definition (Semantics)

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\bar{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

$$
\begin{array}{rcl}
\bar{V}(a, P) & = & V(a, P) \\
\bar{V}(a, i) & = & \begin{cases} 1 & \text{if } a = g(i) \\ 0 & \text{otherwise} \end{cases} \\
\bar{V}(a, \varphi \leq q) & = & \begin{cases} 1 & \text{if } \bar{V}(a, \varphi) \leq q \\ 0 & \text{otherwise} \end{cases} \\
\bar{V}(a, \neg\varphi) & = & 1 - \bar{V}(a, \varphi) \\
\bar{V}(a, \varphi \wedge \psi) & = & \max\{0, \bar{V}(a, \varphi) + \bar{V}(a, \psi) - 1\} \\
\bar{V}(a, \varphi \rightarrow \psi) & = & \min\{1, 1 - \bar{V}(a, \varphi) + \bar{V}(a, \psi)\} \\
\bar{V}(a, \Diamond\varphi) & = & \sum_{b \in A} R(a, b)\bar{V}(b, \varphi) \\
\bar{V}(a, @_i\varphi) & = & \bar{V}(g(i), \varphi) \\
\bar{V}(a, E\varphi) & = & \sup\{\bar{V}(b, \varphi) \mid b \in A\}
\end{array}
$$

# A fuzzy hybrid logic

## Definition (Semantics)

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\bar{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

$$
\begin{aligned}
\bar{V}(a, P) &= V(a, P) \\
\bar{V}(a, i) &= \begin{cases} 1 & \text{if } a = g(i) \\ 0 & \text{otherwise} \end{cases} \\
\bar{V}(a, \varphi \leq q) &= \begin{cases} 1 & \text{if } \bar{V}(a, \varphi) \leq q \\ 0 & \text{otherwise} \end{cases} \\
\bar{V}(a, \neg\varphi) &= 1 - \bar{V}(a, \varphi) \\
\bar{V}(a, \varphi \wedge \psi) &= \max\{0, \bar{V}(a, \varphi) + \bar{V}(a, \psi) - 1\} \\
\bar{V}(a, \varphi \to \psi) &= \min\{1, 1 - \bar{V}(a, \varphi) + \bar{V}(a, \psi)\} \\
\bar{V}(a, \Diamond\varphi) &= \sum_{b \in A} R(a, b)\bar{V}(b, \varphi) \\
\bar{V}(a, @_i\varphi) &= \bar{V}(g(i), \varphi) \\
\bar{V}(a, E\varphi) &= \sup\{\bar{V}(b, \varphi) \mid b \in A\}
\end{aligned}
$$

# A fuzzy hybrid logic

## Definition (Semantics (continued))

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\bar{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

$$\bar{V}(a, [P := \varphi]\psi) = \bar{V}_{P:=\varphi}(a, \psi)$$

$$\bar{V}(a, [P := \Diamond\varphi]^*\psi) = \sup\{\bar{V}(a, [P := \Diamond\varphi]^n\psi) \mid n \in \mathbb{N}\}$$

## Definition (Updated network models)

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \text{PROP}$, and a $\varphi \in \mathcal{L}_D$, we let the updated model $\mathcal{M}_{P:=\varphi}$ be $\langle A, R, g, V_{P:=\varphi} \rangle$, where

$$V_{P:=\varphi}(a, Q) = V(a, Q)\ , \quad \text{for all } Q \in \text{PROP} \setminus \{P\} \text{ and all } a \in A$$

$$V_{P:=\varphi}(a, P) = \bar{V}(a, \varphi)\ , \quad \text{for all } a \in A$$

## Definition (Validity)

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \text{PROP}$, and a formula $\varphi$, $\varphi$ is valid in $\mathcal{M}$ iff $V(a, \varphi) = 1$ for all $a \in A$.

# A fuzzy hybrid logic

**Definition (Semantics (continued))**

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\bar{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

$$\begin{aligned} \bar{V}(a, [P := \varphi]\psi) &= \bar{V}_{P:=\varphi}(a, \psi) \\ \bar{V}(a, [P := \Diamond\varphi]^*\psi) &= \sup\{\bar{V}(a, [P := \Diamond\varphi]^n\psi) \mid n \in \mathbb{N}\} \end{aligned}$$

**Definition (Updated network models)**

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \mathrm{PROP}$, and a $\varphi \in \mathcal{L}_D$, we let the updated model $\mathcal{M}_{P:=\varphi}$ be $\langle A, R, g, V_{P:=\varphi} \rangle$, where

$$\begin{aligned} V_{P:=\varphi}(a, Q) &= V(a, Q)\ , \quad \text{for all } Q \in \mathrm{PROP} \setminus \{P\} \text{ and all } a \in A \\ V_{P:=\varphi}(a, P) &= \bar{V}(a, \varphi)\ , \quad \text{for all } a \in A \end{aligned}$$

**Definition (Validity)**

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \mathrm{PROP}$, and a formula $\varphi$, $\varphi$ is valid in $\mathcal{M}$ iff $V(a, \varphi) = 1$ for all $a \in A$.

# A fuzzy hybrid logic

## Definition (Semantics (continued))

Given a model $\mathcal{M} = \langle A, R, g, V \rangle$, we extend $V$ to $\bar{V}$, for all agents $a \in A$ and all formulas $\varphi \in \mathcal{L}_S$, by the following inductive clauses:

$$\bar{V}(a, [P := \varphi]\psi) = \bar{V}_{P:=\varphi}(a, \psi)$$

$$\bar{V}(a, [P := \Diamond\varphi]^*\psi) = \sup\{\bar{V}(a, [P := \Diamond\varphi]^n\psi) \mid n \in \mathbb{N}\}$$

## Definition (Updated network models)

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \mathrm{PROP}$, and a $\varphi \in \mathcal{L}_D$, we let the updated model $\mathcal{M}_{P:=\varphi}$ be $\langle A, R, g, V_{P:=\varphi} \rangle$, where

$$V_{P:=\varphi}(a, Q) = V(a, Q)\ , \quad \text{for all } Q \in \mathrm{PROP} \setminus \{P\} \text{ and all } a \in A$$

$$V_{P:=\varphi}(a, P) = \bar{V}(a, \varphi)\ , \quad \text{for all } a \in A$$

## Definition (Validity)

For a model $\mathcal{M} = \langle A, R, g, V \rangle$, a $P \in \mathrm{PROP}$, and a formula $\varphi$, $\varphi$ is valid in $\mathcal{M}$ iff $V(a, \varphi) = 1$ for all $a \in A$.

**Properties of the logic**

- $\bar{V}(a, \neg\Diamond\neg\varphi) = \bar{V}(a, \Diamond\varphi)$
- $\bar{V}(a, U\varphi) = \inf\{\bar{V}(b, \varphi) \mid b \in A\}$ $\qquad (U\varphi := \neg E\neg\varphi)$
- $\bar{V}(a, \varphi \to \psi) = 1$ iff $\bar{V}(a, \varphi) \leq \bar{V}(a, \psi)$
- $\bar{V}(a, \varphi \leq q) = 1$ iff $\bar{V}(a, \varphi) \leq q$
- $\bar{V}(a, \varphi = q) = 1$ iff $\bar{V}(a, \varphi) = q$
- $\bar{V}(a, @_i P) = V(g(i), P)$
- $\bar{V}(a, @_i \Diamond j) = R(g(i), g(j))$

# Outline

- DeGroot and Lehrer's model of opinion dynamics
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research

# Outline

- DeGroot and Lehrer's model of opinion dynamics
- A fuzzy hybrid logic
- **Reasoning about opinion dynamics in social networks**
- Concluding remarks and future research

## Formalizing DeGroot's model

- Let $A$ be the set of individuals making up the group
- Let $P$ be the proposition that the agents have opinions about (i.e. PROP $= \{P\}$)
- The initial opinion of agent $a$ towards $P$ is $V(a, P)$
- $R(a, b)$ is $w_{ab}$, i.e. the weight that agent $a$ puts on the opinion of agent $b$
- Choose any function $g$ : NOM $\to A$ and set NOM
- DeGroot's updating mechanism corresponds to a substitution of $P$ by $\Diamond P$.
- I.e. an application of the modality $[P := \Diamond P]$
- $\varphi$ is true after 7 steps of the DeGroot dynamics: $[P := \Diamond P]^7 \varphi$
- That the agents is a consensus of assigning $q$ to $P$: $(UP) = q$.
- The agents are in a consensus: $EP \to UP$

**Formalizing DeGroot's model**

- Let $A$ be the set of individuals making up the group
- Let $P$ be the proposition that the agents have opinions about (i.e. $\text{PROP} = \{P\}$)
- The initial opinion of agent $a$ towards $P$ is $V(a, P)$
- $R(a, b)$ is $w_{ab}$, i.e. the weight that agent $a$ puts on the opinion of agent $b$
- Choose any function $g : \text{NOM} \to A$ and set NOM
- DeGroot's updating mechanism corresponds to a substitution of $P$ by $\Diamond P$.
- I.e. an application of the modality $[P := \Diamond P]$
- $\varphi$ is true after 7 steps of the DeGroot dynamics: $[P := \Diamond P]^7 \varphi$
- That the agents is a consensus of assigning $q$ to $P$: $(UP) = q$.
- The agents are in a consensus: $EP \to UP$

**Formalizing DeGroot's model**

- Let $A$ be the set of individuals making up the group
- Let $P$ be the proposition that the agents have opinions about (i.e. $\text{PROP} = \{P\}$)
- The initial opinion of agent $a$ towards $P$ is $V(a, P)$
- $R(a, b)$ is $w_{ab}$, i.e. the weight that agent $a$ puts on the opinion of agent $b$
- Choose any function $g : \text{NOM} \to A$ and set NOM
- DeGroot's updating mechanism corresponds to a substitution of $P$ by $\Diamond P$.
- I.e. an application of the modality $[P := \Diamond P]$
- $\varphi$ is true after 7 steps of the DeGroot dynamics: $[P := \Diamond P]^7 \varphi$
- That the agents is a consensus of assigning $q$ to $P$: $(UP) = q$.
- The agents are in a consensus: $EP \to UP$

## Formalizing DeGroot's model

- The agents are in a consensus: $EP \to UP$
- If consensus is reached after a finite number of steps $n$, then $\bar{V}(a, [P := \Diamond P]^*(EP \to UP)) = 1$.
- However, the other doesn't quite work
- Moreover, it does not guarantee convergence to a consensus

**Theorem**

*Given a network model $\mathcal{M}$. Then, a consensus is reached in $\mathcal{M}$ under the DeGroot dynamics (i..e successive updates with $[P := \Diamond P]$), if and only if, the formula*

$$[P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)$$

*is valid in $\mathcal{M}$.*

The proof uses that: $\bar{V}(a, [P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)) = \liminf_{n\to\infty} \bar{V}(a, [P := \Diamond P]^n (EP \to UP))$

- properties in the limit can be expressed as: $[P := \Diamond P]^* \langle P := \Diamond P \rangle^* \varphi$

## Formalizing DeGroot's model

- The agents are in a consensus: $EP \to UP$
- If consensus is reached after a finite number of steps $n$, then $\bar{V}(a, [P := \Diamond P]^*(EP \to UP)) = 1$.
- However, the other doesn't quite work
- Moreover, it does not guarantee convergence to a consensus

**Theorem**

*Given a network model $\mathcal{M}$. Then, a consensus is reached in $\mathcal{M}$ under the DeGroot dynamics (i..e successive updates with $[P := \Diamond P]$), if and only if, the formula*

$$[P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)$$

*is valid in $\mathcal{M}$.*

The proof uses that: $\bar{V}(a, [P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)) = \liminf_{n \to \infty} \bar{V}(a, [P := \Diamond P]^n (EP \to UP))$

- properties in the limit can be expressed as: $[P := \Diamond P]^* \langle P := \Diamond P \rangle^* \varphi$

### Formalizing DeGroot's model

- The agents are in a consensus: $EP \to UP$
- If consensus is reached after a finite number of steps $n$, then $\bar{V}(a, [P := \Diamond P]^*(EP \to UP)) = 1$.
- However, the other doesn't quite work
- Moreover, it does not guarantees convergence to a consensus

**Theorem**

*Given a network model $\mathcal{M}$. Then, a consensus is reached in $\mathcal{M}$ under the DeGroot dynamics (i..e successive updates with $[P := \Diamond P]$), if and only if, the formula*

$$[P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)$$

*is valid in $\mathcal{M}$.*

The proof uses that: $\bar{V}(a, [P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)) = \liminf_{n \to \infty} \bar{V}(a, [P := \Diamond P]^n (EP \to UP))$

- properties in the limit can be expressed as: $[P := \Diamond P]^* \langle P := \Diamond P \rangle^* \varphi$

## Formalizing DeGroot's model

- The agents are in a consensus: $EP \to UP$
- If consensus is reached after a finite number of steps $n$, then $\bar{V}(a, [P := \Diamond P]^*(EP \to UP)) = 1$.
- However, the other doesn't quite work
- Moreover, it does not guarantee convergence to a consensus

**Theorem**

*Given a network model $\mathcal{M}$. Then, a consensus is reached in $\mathcal{M}$ under the DeGroot dynamics (i..e successive updates with $[P := \Diamond P]$), if and only if, the formula*

$$[P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)$$

*is valid in $\mathcal{M}$.*

The proof uses that: $\bar{V}(a, [P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)) = \liminf_{n \to \infty} \bar{V}(a, [P := \Diamond P]^n (EP \to UP))$

- properties in the limit can be expressed as: $[P := \Diamond P]^* \langle P := \Diamond P \rangle^* \varphi$

## Formalizing DeGroot's model

- The agents are in a consensus: $EP \to UP$
- If consensus is reached after a finite number of steps $n$, then $\bar{V}(a, [P := \Diamond P]^*(EP \to UP)) = 1$.
- However, the other doesn't quite work
- Moreover, it does not guarantees convergence to a consensus

**Theorem**

*Given a network model $\mathcal{M}$. Then, a consensus is reached in $\mathcal{M}$ under the DeGroot dynamics (i..e successive updates with $[P := \Diamond P]$), if and only if, the formula*

$$[P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)$$

*is valid in $\mathcal{M}$.*

The proof uses that: $\bar{V}(a, [P := \Diamond P]^* \langle P := \Diamond P \rangle^* (EP \to UP)) = \liminf_{n \to \infty} \bar{V}(a, [P := \Diamond P]^n (EP \to UP))$

- properties in the limit can be expressed as: $[P := \Diamond P]^* \langle P := \Diamond P \rangle^* \varphi$

# Outline

- DeGroot and Lehrer's model of opinion dynamics
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- Concluding remarks and future research

# Outline

- DeGroot and Lehrer's model of opinion dynamics
- A fuzzy hybrid logic
- Reasoning about opinion dynamics in social networks
- **Concluding remarks and future research**

# Concluding remarks and future research

## Conclusions

- Modal logic is suited for reasoning about simple social network dynamics
- More complex dynamics such as DeGroot's can also be captured in proper extensions of modal logic
- **MY QUEST:**

**LOGICAL METHODS FOR SOCIAL NETWORK ANALYSIS**

## Future research

- Develop a proof theory or decision procedure for the logic
- Compare to probabilistic logics
- Compare to logics for Markov chains etc.

# Concluding remarks and future research

**Conclusions**

- Modal logic is suited for reasoning about simple social network dynamics
- More complex dynamics such as DeGroot's can also be captured in proper extensions of modal logic
- **MY QUEST:**

**LOGICAL METHODS FOR SOCIAL NETWORK ANALYSIS**

**Future research**

- Develop a proof theory or decision procedure for the logic
- Compare to probabilistic logics
- Compare to logics for Markov chains etc.

# Concluding remarks and future research

**Conclusions**

- Modal logic is suited for reasoning about simple social network dynamics
- More complex dynamics such as DeGroot's can also be captured in proper extensions of modal logic
- **MY QUEST:**

**LOGICAL METHODS FOR SOCIAL NETWORK ANALYSIS**

**Future research**

- Develop a proof theory or decision procedure for the logic
- Compare to probabilistic logics
- Compare to logics for Markov chains etc.